Quantum Entropy Scoring for Fast Robust Mean Estimation and Outlier Detection

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**Problem setup**
- Given a distribution $D$ over $\mathbb{R}^d$ with mean $\mu$, let $X \in \mathbb{R}^{n \times d}$ be $n$ i.i.d. samples from $D$, with $\epsilon$-fraction of them corrupted, efficiently detect the corrupted data and estimate the sample mean $\hat{\mu}$, with good error bound.
- This is nontrivial: Naïve estimates have errors that scale with dimension.
- Many applications: robust regression; detecting fraud, medical anomalies, network traffic irregularities; etc.

**Prior Work**
- Naïve spectral: filtering method based on projection onto top eigenvector, $O(nd^2)$ complexity.
- Best prior result:
  - $O(\min(n d^2, n d \epsilon n^6))$ time complexity.
  - $d$-independent error bound: $\|\mu - \hat{\mu}\|_2 \leq O(\epsilon)$.
- Collective inductive bias can be detected by the spectra.

**Our contribution**
- **QUE-scoring**: nearly linear time complexity $O(nd)$.
- $d$-independent error bound.
- **QUE interpolates** between scores based on $l_2$-norm and projection onto the top eigenvalue, controlled by $\alpha$.
  - Inspecting multiple directions at once.
  - $\text{QUE}(X) = (X_i - E[X])^T U (X_i - E[X])$, $U = \frac{\exp(\alpha \text{cov}(X))}{\text{trace}(\exp(\alpha \text{cov}(X)))}$
- Lower $\alpha$ -> like $l_2$, higher $\alpha$ -> like naïve spectral.
- **Fast computation** possible by combining
  - fast Johnson-Lindenstrauss.
  - Chebyshev expansion of $\exp(\text{cov}(X))$.
  - fast Hadamard transform.
- Works well in high dimensions

**Experiments: datasets**
- Synthetic: i.i.d. samples from $N(0, I_d)$.
- Outliers: i.i.d. samples from mixture of Gaussians $N(\frac{\mu}{\epsilon}, \sigma^2 I_d)$.
- Text: i.i.d. samples from sections of Sherlock Holmes.
- Outliers: word embeddings of Wikipedia articles.
- Image: i.i.d. samples from CIFAR images.
- Outliers: images with corrupted pixels

**Experiments: Results**
- QUE scoring and baselines v.s. number of outlier directions.
- QUE scoring improvement over naïve spectral scoring w.r.t. $\alpha$.
- QUE scoring improvement over $l_2$ scoring w.r.t. $\alpha$.