Problem Set 3

October 23, 2024

Due November 8

Problem 1: SoS Proof for Clique Size Bound

Let G be a graph drawn from $G(n, 1/2)$. Show that with high probability, there exists a sum-of-squares (SoS) proof of constant degree that certifies that G does not contain any clique of size greater than $O(\sqrt{n \log n})$.

Hint: first construct a system of polynomials whose solutions correspond to cliques in G. The variables should be $0/1$ indicators for the presence of each vertex in a clique.

Problem 2: Robustness to Adversarial Modifications

Suppose a malicious adversary is allowed to modify any subset of $n^{0.99}$ edges of a graph drawn from $G(n, 1/2)$. Show that, despite this, there still exists with high probability a constant-degree SoS proof that certifies the graph does not contain any clique of size greater than $O(\sqrt{n \log n})$.

Problem 3: Planted 2-XOR

Let ϕ be a random instance of $2 - XOR$ over $\{\pm 1\}$, sampled in the following way. First, choose $x^* \in \{\pm 1\}^n$. Then, for each $(i, j) \in [n]^2$, with probability $(Cn \log n)/n^2$:

1. with probability 0.99, add the constraint $x_i x_j = x_i^* x_j^*$ to ϕ
2. otherwise (hence, with probability 0.01) add the constraint $x_i x_j = -x_i^* x_j^*$ to ϕ.

The resulting instance ϕ will have about $Cn \log n$ equations.

Part a: show that for large-enough C, w.h.p. there exists $y \in \{\pm 1\}^n$ which satisfies 98% of the equations in ϕ.

Hint: choose $y = x^*$

Part b: again for large-enough C, show that there is a polynomial-time algorithm which finds some $y \in \{\pm 1\}^n$ which satisfies at least 97% of the equations in ϕ, with high probability over the choice of ϕ.

Hint: first show that you can round a pseudoexpectation to find y which is correlated with x^*